CALCULUS 2

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دفتر الطالبة :

فرح المحاسنه

بسم الله الرحمن الرحيم

CH 7

Techniques of Integration

[1] Integration by parts.

Given 2 func. u and V of x, then

$$d(uv) = udv + vdu$$

* $udv = duv - vdu$

$$\int udv = \int d(uv) - \int vdu$$

فرحة *_*

$$= \boxed{\int udv = uv - \int vdu}$$

Ex

$$\int x e^x dx$$

$u = x$ $\qquad$ $dv = e^x dx$

$du = dx$ $\qquad$ $v = e^x$

$$= xe^x - \int e^x dx$$

$$= xe^x - e^x + C$$

Ex

$$\int \underbrace{x^2}_{u} \, \underbrace{e^x \, dx}_{dv}$$

$$= x^2 e^x - \int 2e^x \, x \, dx$$

$$= x^2 e^x - 2(e^x - e^x) + C$$

Ex

$$\int \underbrace{\tan^{-1} x}_{u} \, \underbrace{dx}_{dv}$$

$du = \frac{1}{1+x^2}$ $\qquad$ $v = x$

$$= \tan^{-1} x \, (x) - \int \frac{x}{1+x^2} \, dx$$

$z = 1 + x^2$

$dz = 2x \, dx$

$\frac{dz}{2} = x \, dx$

$$= x \tan^{-1} x - \int \frac{1}{2z} \, dz$$

$$= x \tan^{-1} x - \frac{1}{2} \ln |z| + C$$

$$= x \tan^{-1} x - \frac{1}{2} \ln |1 + x^2| + C$$

صلّي على الحبيب <3

Ex

$$\int \underset{u}{\ln x}\ \underset{dv}{dx}$$

لا تنسى لكتابة C !!

$du = \frac{1}{x}$ $\qquad v = x$

$= x \ln x - \int \frac{x}{x}\, dx$

$= x \ln x - x + C$

Ex

$$\int \underset{u}{\ln(1+x^2)}\ \underset{dv}{dx}$$

$du = \frac{2x}{1+x^2}$ $\qquad v = x$

$= x \ln(1+x^2) - \int x \left(\frac{2x}{1+x^2}\right) dx$

$= x \ln(1+x^2) - \int \frac{2x^2}{1+x^2}\, dx$

نقسم

$$\begin{array}{r} 2 \\ x^2+1 \,\big)\, 2x^2 \\ -2x^2 + 2 \\ \hline -2 \end{array}$$

$= x \ln(1+x^2) - \int 2 - \frac{2}{1+x^2}\, dx$

$= x \ln(1+x^2) - 2x + \tan^{-1} x + C$

Ex

$$\int \sec^3 x \, dx$$

$$\int \underbrace{\sec x}_{u} \; \underbrace{\sec^2 x \, dx}_{dv}$$

$du = \sec x \tan x \, dx \qquad v = \tan x$

$$= \sec x \tan x - \int \tan x \sec x \tan x \, dx$$

$$= \sec x \tan x - \int \tan^2 x \sec x \, dx$$

$$= \sec x \tan x - \int \sec x \, (\sec^2 x - 1) \, dx$$

$$= \sec x \tan x - \int \sec^3 x - \sec x \, dx$$

$$I = \sec x \tan x - \underbrace{\int \sec^3 x \, dx}_{I} + \int \sec x \, dx$$

$$2I = \sec x \tan x + \int \sec x \, dx \left(\frac{\sec x + \tan x}{\sec x + \tan x} \right)$$

$$2I = \sec x \tan x + \ln|\sec x + \tan x|$$

$$I = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C$$

The end

Ex $\int \underbrace{e^{2x}}_{u} \underbrace{\cos x\, dx}_{dv}$

$du = 2e^{2x}$ $\quad v = \sin x$

$= e^{2x} \sin x - \int \overbrace{\sin x}^{dv} \underbrace{(2e^{2x})}_{u} dx$

$= e^{2x} \sin x - \left[-2e^{2x} \cos x - \int -\cos x \, (4e^{2x}) \cdot dx \right.$

$I = e^{2x} \sin x + 2e^{2x} \cos x - 4 \int \underbrace{\cos x \, e^{2x} dx}_{I}$

$5I = e^{2x} \sin x + 2e^{2x} \cos x$

$I = \frac{1}{5} e^{2x} \sin x + \frac{2}{5} e^{2x} \cos x + C$

\# $\int e^{x} \cosh x \quad dx$

Coshx نعوض قيمة . لايجوز بالأجزاء

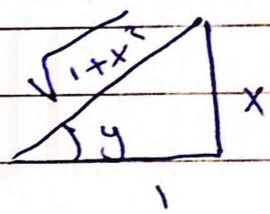
Ex $\int \frac{\tan^{-1}x}{(1+x^2)^{3/2}}\, dx$

let $y = \tan^{-1}x$ $\longrightarrow$ $x = \tan y$

$dy = \frac{1}{1+x^2}\, dx$

$\int \frac{y}{(1+x^2)^{\frac{1}{2}}}\, dy = \int \frac{y}{(1+\tan^2 y)^{\frac{1}{2}}}\, dy = \int \frac{y}{|\sec y|}\, dy$

$\int \underbrace{y}_{u} \; \underbrace{\cos y\, dx}_{dv}$



$= y \sin y - \int \sin y\, dy$

$= y \sin y + \cos y + C$

$= \tan^{-1}x \left(\frac{x}{\sqrt{1+x^2}}\right) + \frac{1}{\sqrt{1+x^2}} + C$

Ex $\int \frac{x e^x}{(1+x)^2}\, dx$

$u = x e^x \qquad dv = \frac{1}{1+x^2}\, dx$

$du = x e^x + e^x \; dx \qquad v = \frac{-1}{1+x}$

$= e^x(x+1)$

$= \frac{-x e^x}{1+x} - \int - \frac{e^x (\cancel{x+1})}{\cancel{1+x}}\, dx$

$= \frac{-x e^x}{1-x} + e^x + C$ ##

*** Trigonometric Integrals** * التكاملات المثلثية *

[1] $\int \sin x \, dx = -\cos x + C$

[2] $\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2}\, dx =$

$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + C$

$= \frac{1}{2} x - \frac{1}{2} \sin x \cos x + C$

تذكر : **

$\cos 2x = 1 - 2\sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

#

$$I_n = \int \sin^n x \, dx$$ n is apositive $n \geq 2$

$$= \int \underbrace{\sin^{n-1} x}_{u} \underbrace{\sin x \, dx}_{dv}$$

$$= \sin^{n-1} x (-\cos x) - \int -\cos x \, (n-1) \sin^{n-2} x \cos x \, dx$$

$$= -\sin^{n-1} x \cos x + \int (n-1) \sin^{n-2} x \cos^2 x \, dx$$

$$= -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x (1 - \sin^2 x) \, dx$$

$$I_n = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x - (n-1) \underbrace{\int \sin^n x \, dx}_{I_n}$$

$$I_n + (n-1) I_n = -\sin^{n-1} x \cdot \cos x + (n-1) \underbrace{\int \sin^{n-2} x \, dx}_{I_{n-2}}$$

$$n I_n = -\sin^{n-1} x \cos x + (n-1) I_{n-2}$$

$$\boxed{I_n = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2}}$$

This is call the reduction formula for $I_n = \int \sin^n x \, dx$

Ex Find $I_6 = \int \sin^6 x \, dx$

$$= -\frac{1}{6}\sin^5 x \cos x + \frac{5}{6} I_4$$

$$= -\frac{1}{6}\sin^5 x \cos x + \frac{5}{6}\left[-\frac{1}{4}\sin^3 x \cos x + \frac{3}{4} I_2\right]$$

$$= -\frac{1}{6}\sin^5 x \cos x - \frac{5}{24}\sin^3 x \cos x + \frac{15}{24}\left[-\frac{1}{2}\sin x \cos x + \frac{1}{2}x\right]$$

\#

$$I_n = \int \cos^n x \, dx$$

$$\boxed{I_n = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}}$$

نفس طريقة الاشتقاق ^^

* $\int \sin^{\frac{5}{2}} x \, \cos^3 x \, dx$

$= \int \sin^{\frac{5}{2}} x \, \cos^2 x \, \cos x \, dx$

$= \int \sin^{\frac{5}{2}} x \, (1-\sin^2 x) \cos x \, dx$

$= \int y^{\frac{5}{2}} (1-y^2) \, dy$ $\qquad y = \sin x$

$= \int y^{\frac{5}{2}} \, dy - \int y^{\frac{9}{2}} \, dy$ $\qquad dy = \cos x \, dx$

$= \frac{2}{7} y^{\frac{7}{2}} - \frac{2}{11} y^{\frac{11}{2}} + C$

$= \frac{2}{7} (\sin x)^{\frac{7}{2}} - \frac{2}{11} (\sin x)^{\frac{11}{2}} + C$

هدف بلا مجهود = أحلام

مجهود بلا أهداف = وقت ضائع

هدف + مجهود = تغيير العالم

$$\int \cos^6 x \, \sin^4 x \, dx$$

$$= \int \cos^6 x \, (1-\cos^2 x)^2 \, dx$$

$$= \int \cos^6 x \, (1 - 2\cos^2 x + \cos^4 x) \, dx$$

$$= \int \cos^{10} x \, dx - 2\int \cos^8 x \, dx + \int \cos^6 x \, dx$$

$$= \frac{1}{10} \cos^9 x \sin x - \frac{9}{10} I_8 + 2 I_8 + \int \cos^6 x \, dx$$

$$= \frac{1}{10} \cos^9 x \sin x - \frac{11}{10} I_8 + \int \cos^6 x \, dx$$

$$= \frac{1}{10} \cos^9 x \sin x - \frac{11}{10} \left(\frac{1}{8} \cos^7 x \sin x + \frac{7}{8} I_6 \right) + I_6$$

$$= \frac{1}{10} \cos^9 x \sin x - \frac{11}{80} \cos^7 x \sin x - \frac{77}{80} I_6 + I_6$$

$$= \frac{1}{10} \cos^9 x \sin x - \frac{11}{80} \cos^7 x \sin x + \frac{3}{80} \left[\frac{1}{6} \cos^5 x \sin x + \frac{5}{6} I_4 \ldots \right.$$

⋮

نكمل على نفس الطريقة :

* $\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = -\ln|\cos x| + C$

$= \ln|\sec x| + C$

* $\int \tan^2 x \, dx = \int \sec^2 x - 1 \, dx$

$= \tan x - x + C$

* $\int \underbrace{x}_{u} \, \underbrace{\tan^2 x \, dx}_{dv} \rightsquigarrow$ by parts

$= x(\tan x - x) - \int \tan x - x \, dx$

$= x\tan x - x^2 - \ln|\sec x| + \frac{x^2}{2} + C$

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\# $I_n = \int \tan^n x \, dx \qquad , n \geq 2$ positive integer

$$= \int \tan^{n-2} x \tan^2 x \, dx$$

$$= \int \tan^{n-2} x \, (\sec^2 x - 1) \, dx$$

$$= \int \tan^{n-2} x \sec^2 x \, dx - \underbrace{\int \tan^{n-2} x \, dx}_{I_{n-2}}$$

$$= \int y^{n-2} \, dy - I_{n-2}$$

$y = \tan x$

$dy = \sec^2 x \, dx$

$$I_n = \frac{y^{n-2}}{n-1} - I_{n-2}$$

$$\boxed{I_n = \frac{1}{n-1} \tan^{n-2} x - I_{n-2}}$$

\# $I_5 = \int \tan^5 x \, dx$

$$= \frac{1}{4} \tan^4 x - I_3$$

$$= \frac{1}{4} \tan^4 x - \frac{1}{2} \tan x + I$$

$$= \frac{1}{4} \tan^4 x - \frac{1}{2} \tan x - \ln|\cos x| + c$$

\# $\int \sec x \, dx$

$= \ln|\sec x + \tan x| + C$

* $\int \sec^2 x \, dx = \tan x + C$

\# $I_n = \int \sec^n x \, dx \quad ; n \geq 3$

$= \int \underbrace{\sec^{n-2} x}_{u} \underbrace{\sec^2 x \, dx}_{dv}$ ~> by parts

$= \sec^{n-2} x \tan x - \int \tan x \,(n-2) \sec^{n-3} x \sec x \tan x \, dx$

$= \sec^{n-2} x \tan x - (n-2) \int \sec^{n-2} x \tan^2 x \, dx$

$= \sec^{n-2} x \tan x - (n-2) \int \sec^{n-2} x (\sec^2 x - 1) \, dx$

$I_n = \sec^{n-2} x \tan x - (n-2) \underbrace{\int \sec^n x \, dx}_{I_n} + (n-2) \underbrace{\int \sec^{n-2} x \, dx}_{I_{n-2}}$

$I_n + (n-2) I_n = \sec^{n-2} x \tan x + \cancel{(n-2) \int \sec^{(n-2)} x}\, I_{n-2}$

$$\boxed{I_n = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} I_{n-2}}$$ \#

ضَعْ بصمتك في حياتك

* $\int \tan^7 x \sec^4 x \, dx$

1- $\int \tan^7 x \sec^2 x \sec^2 x \, dx$

$\int \tan^7 x \, (\tan^2 x + 1) \sec^2 x \, dx$

$y = \tan x$
$dy = \sec^2 x \, dx$

$\int y^7 (y^2 + 1) \, dy$

$\int y^9 \, dy + \int y^7 \, dy$

$\frac{y^{10}}{10} + \frac{y^8}{8} + c$

$\frac{\tan^{10} x}{10} + \frac{\tan^8 x}{8} + c$

OK

2- $\int \tan^6 x \sec^3 x \tan x \sec x \, dx$

$= \int (\sec^2 x - 1)^3 \sec^3 x \sec x \tan x \, dx$

$= \int (y^2 - 1)^3 \, y^3 \, dy$

$y = \sec x$
$dy = \sec x \tan x \, dx$

$\int (y^6 - 3y^7 + 3y^2 - 1) \, y^3 \, dy$

$\int y^9 - 3y^{10} + 3y^5 - y^3 \, dy$

$= \frac{y^{10}}{10} - \frac{3y^{11}}{11} + \frac{3y^6}{6} - \frac{y^4}{4} + c$

ونرجع للتعويض

Five Apple

$$\int \sec^3 x \tan^6 x \, dx$$

$$= \int \sec^3 x \,(\sec^2 x - 1)^3 \, dx$$

$$\int \sec^3 x \,(\sec^6 x - 3\sec^2 x + 3\sec^2 x - 1)\, dx$$

$$= \int \sec^9 x \, dx - 3\int \sec^7 x \, dx + 3\int \sec^5 x \, dx - \int \sec^3 x \, dx$$

ö Continue :

Ex $\displaystyle \int \frac{\sin(2x)}{1+\sin x}\, dx$

$$= \int \frac{2 \sin x \cos x}{1+\sin x}\, dx$$

$y = \sin x$

$dy = \cos x \, dx$

$$= \int \frac{2y \, dy}{1+y}$$

$$\begin{array}{r} 2 \\ y+1 \,\big)\, 2y \\ -(2y + 2) \\ \hline -2 \end{array}$$

$$= \int 2 - \frac{2}{1+y}\, dy$$

$$= 2y - 2\ln|1+y| + C$$

$$= 2\sin x - 2\ln|1+\sin x| + C$$

* $\int \sin(10x) \cos(4x)\, dx$

$= \int \frac{1}{2}\left[\sin(6x) + \sin(14x)\right] dx$

$\frac{1}{2}\left[-\frac{1}{6}\cos 6x - \frac{1}{14}\cos(14x)\right] + C$

Remember :

- $\sin\alpha \cos\beta =$
$\frac{1}{2}\left[\sin(\alpha-\beta) + \sin(\alpha+\beta)\right]$

* $\int \sin(3x) \sin(10x)\, dx$

$= \int \frac{1}{2}\left[\cos(-7x) - \cos(13x)\right] dx$

$\frac{1}{2}\left[\frac{1}{7}\sin(7x) - \frac{1}{13}\sin(13x)\right] + C$

- $\sin\alpha \sin\beta =$
$\frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$

* $\int \cos(3x) \cos(2x)\, dx$

$= \int \frac{1}{2}\left[\cos x + \cos(5x)\right] dx$

$= \frac{1}{2}\left[\sin x + \frac{1}{5}\sin(5x)\right] + C$

- $\cos\alpha \cos\beta =$
$\frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$

Section 3

Trigeonetric Subsitions.

$$\int \frac{x^2\,dx}{\sqrt{4-x^2}}$$

* لو كان هناك سؤال فيه بالمقام

$a^2 - u^2$

ثابت — متغير

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

نفرض

$u = a\sin\theta$

$$= \int \frac{4\sin^2\theta}{\sqrt{4-4\sin^2\theta}} * 2\cos\theta\, d\theta$$

$\theta = \sin^{-1}\left(\frac{u}{a}\right)$

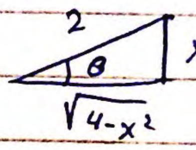
$$= \int \frac{4\sin^2\theta\; 2\cos\theta\, d\theta}{2\sqrt{1-\sin^2\theta}} = \int \frac{8\sin^2\theta\, \cancel{\cos\theta}}{2\,\cancel{\cos\theta}}\, d\theta$$

$$= \int 4\sin^2\theta\, d\theta$$

$$= 4\int \frac{1-\cos 2\theta}{2}\, d\theta$$

$\sin 2\theta \Rightarrow 2\sin\theta\cos\theta$

$$2\left(\theta - \frac{\sin 2\theta}{2}\right) + C$$



$$= 2\left(\sin^{-1}\left(\frac{x}{2}\right) - \frac{x}{2}\,\frac{\sqrt{4-x^2}}{2}\right) + C$$

* اذا كانت على شكل
a^2+u^2
$u = a\tan\theta$

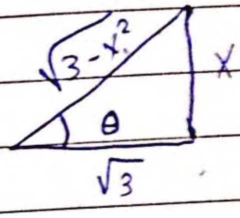
* $\int \sqrt{3+x^2}\,dx$

$x=\sqrt{3}\tan\theta \quad \to \quad \tan\theta = \frac{x}{\sqrt{3}}$

$dx = \sqrt{3}\sec^2\theta\, d\theta$

$= \int \sqrt{3+3\tan^2\theta}\;(\sqrt{3}\sec^2\theta)\,d\theta$

$= \int \sqrt{3}\sec\theta\;\sqrt{3}\sec^2\theta\,d\theta$



$= 3\int \sec^3\theta\,d\theta$

$= 3\left(\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta|\right)+C$

$= 3\left(\frac{1}{2}\frac{\sqrt{3-x^2}}{\sqrt{3}}\left(\frac{x}{\sqrt{3}}\right) + \frac{1}{2}\ln\left|\frac{\sqrt{3-x^2}}{\sqrt{3}} + \frac{x}{\sqrt{3}}\right|\right)+C$

* اذا كانت u^2-a^2
$u = a\sec\theta$

* $\int \frac{\sqrt{x^2-4^2}}{x}\,dx$

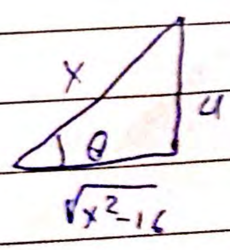
$x = 4\sec\theta$

$dx = 4\sec\theta\tan\theta\,d\theta$

$= \int \frac{\sqrt{4^2\sec^2\theta-4^2}}{x}\;(4\sec\theta\tan\theta)\,d\theta$

$= \int \frac{4\tan\theta}{\cancel{4\sec\theta}}\;(4\cancel{\sec\theta}\tan\theta)\,d\theta$

$= \int 4\tan^2\theta\,d\theta$



$= 4\int \sec^2\theta - 1\,d\theta$

$4(\tan\theta - \theta)+C$

$= 4\left(\frac{4}{\sqrt{x^2-16}} - \sec^{-1}\left(\frac{x}{4}\right)\right)+C$

$$\int \frac{x^2+4}{\sqrt{x^2+2x}}\,dx \rightsquigarrow$$ اكمال مربع

$$x^2+2x+1-1$$
$$(x+1)^2-1$$

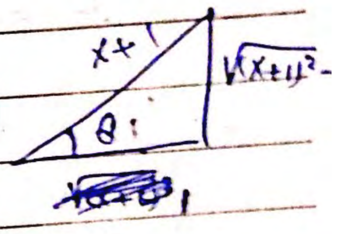
$$=\int \frac{x^2+4}{\sqrt{(x+1)^2-1}}\,dx$$

$x+1=\sec\theta$

$dx=\sec\theta\tan\theta\,d\theta$

$$=\int \frac{(\sec\theta-1)^2+4}{\cancel{\sqrt{\sec^2\theta-1}}}(\sec\theta\,\cancel{\tan\theta})\,d\theta$$

$$\int(\sec^2\theta-2\sec\theta+5)\sec\theta\,d\theta$$



$$=\int\sec^3\theta-2\int\sec^2\theta+5\int\sec\theta$$

$$=\frac{1}{2}\sec\theta\tan\theta+\frac{1}{2}\ln|\sec\theta+\tan\theta|-2\tan\theta+5\ln|\sec\theta+\tan\theta|+C$$

$$=\frac{1}{2}\sec\theta\tan\theta+\frac{11}{2}\ln|\sec\theta+\tan\theta|-2\tan\theta+C$$

$$=\frac{1}{2}(x+1)\left(\sqrt{x^2+2x}\right)+\frac{11}{2}\ln\left|(x+1)+\sqrt{x^2+2x}\right|-2\sqrt{x^2+2x}+C$$

$$* \int \frac{x\,dx}{\sqrt{x-4x^2+1}}$$

$x-4x^2+1 \longrightarrow$ إكمال مربع

$$-4\left(x^2-\frac{1}{4}x-\frac{1}{4}+\frac{1}{64}-\frac{1}{64}\right)$$

$$-4\left(\left(x-\frac{1}{8}\right)^2-\frac{17}{64}\right)$$

$$\frac{17}{16}-4\left(x-\frac{1}{8}\right)^2$$

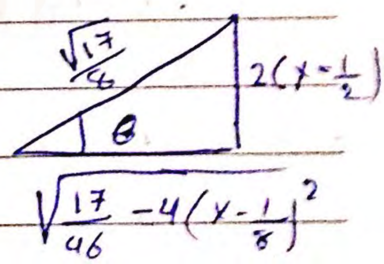
$$\frac{17}{16}-\left(2\left(x-\frac{1}{8}\right)\right)^2$$

$$2\left(x-\frac{1}{8}\right)=\frac{\sqrt{17}}{4}\sin\theta$$

$$2\,dx=\frac{\sqrt{17}}{4}\cos\theta\,d\theta$$

$$=\int \frac{\left(\frac{\sqrt{17}}{8}\sin\theta+\frac{1}{8}\right)\frac{\sqrt{17}}{8}\cos\theta\,d\theta}{\sqrt{\frac{17}{16}-\frac{17}{16}\sin^2\theta}}$$

$$=\int \frac{\left(\frac{\sqrt{17}}{8}\sin\theta+\frac{1}{8}\right)\frac{\cancel{\sqrt{17}}}{\cancel{8}_{2}}\cancel{\cos\theta}\,d\theta}{\frac{\cancel{\sqrt{17}}}{\cancel{4}}\cancel{\cos\theta}}$$



$$=\frac{1}{2}\int \frac{\sqrt{17}}{8}\sin\theta+\frac{1}{8}\;d\theta$$

$$=\frac{1}{2}\left[\frac{\sqrt{17}}{8}(-\cos\theta)+\frac{1}{8}\theta\right]+C$$

نعوض θ بدلالة x

$$\int \frac{dx}{e^{2x}+e^{x}+1}$$

$$e^{x}=y$$

$$dy=e^{x}dx \longrightarrow dx=\frac{dy}{e^{x}}$$

$$=\int \frac{dy}{y(y^2+y+1)} \rightsquigarrow \text{نكامل}$$

$$y^2+y+1=y^2+y+\frac{1}{4}-\frac{1}{4}+1$$

$$=\left(y+\frac{1}{2}\right)^2+\frac{3}{4}$$

$$y+\frac{1}{2}=\frac{\sqrt{3}}{2}\tan\theta$$

$$dy=\frac{\sqrt{3}}{2}\sec^2\theta\, d\theta$$

$$=\int \frac{\frac{\sqrt{3}}{2}\sec^2\theta\, d\theta}{\left(\frac{\sqrt{3}}{2}\tan\theta-\frac{1}{2}\right)\left(\frac{3}{4}\tan^2\theta+\frac{3}{4}\right)}$$

$$=\frac{2}{\sqrt{3}}\int \frac{d\theta}{\frac{\sqrt{3}}{2}\tan\theta-\frac{1}{2}}$$

$$=\frac{2}{\sqrt{3}}\int \frac{d\theta}{\frac{\sqrt{3}}{2}\frac{\sin\theta}{\cos\theta}-\frac{1}{2}} \qquad =\frac{2}{\sqrt{3}}\int \frac{\cos\theta\, d\theta}{\frac{\sqrt{3}}{2}\sin\theta-\frac{\cos\theta}{2}}$$

$$=\frac{2}{\sqrt{3}}\int \frac{\cos\theta\, d\theta}{\sin\theta\,\frac{\sqrt{3}}{2}-\frac{1}{2}\cos\theta} \rightsquigarrow \sin(\alpha-\beta)\ \text{مطابقة}$$

$$\longrightarrow$$

$$= \frac{2}{\sqrt{3}} \int \frac{\cos\theta \, d\theta}{\sin\left(\theta - \frac{\pi}{6}\right)}$$

$u = \theta - \frac{\pi}{6}$

$du = d\theta$

$\theta = u + \frac{\pi}{6}$

$$= \frac{2}{\sqrt{3}} \int \frac{\cos\left(u + \frac{\pi}{6}\right)}{\sin u} \, du$$

$$= \frac{2}{\sqrt{3}} \int \frac{\cos u \cos\frac{\pi}{6} - \sin u \sin\frac{\pi}{6}}{\sin u} \, du$$

$$= \frac{2}{\sqrt{3}} \int \cot u \cos\frac{\pi}{6} - \sin\frac{\pi}{6} \, du$$

$$= \frac{2}{\sqrt{3}} \int \cot u \left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2} \, du$$

$$= \frac{2}{\sqrt{3}} \left[\frac{\sqrt{3}}{2} \ln|\sin u| - \frac{1}{2} u \right] + C$$

$$= \frac{2}{\sqrt{3}} \left[\frac{\sqrt{3}}{2} \ln\left|\sin\left(\theta - \frac{\pi}{6}\right)\right| - \frac{1}{2}\left(\theta - \frac{\pi}{6}\right) \right] + C$$

ونعوض قيمة θ بدلالة y ونعوض y بدلالة x ..

* partial fractions # الكسور الجزئية

Consider $\frac{1}{x-1} + \frac{2}{x+1} = \frac{x+1+2(x-1)}{(x-1)(x+1)}$

$$= \frac{3x-1}{(x-1)(x+1)}$$

$\frac{1}{x-1}$ and $\frac{2}{x+1}$ are called partial fraction for $\frac{3x-1}{(x-1)(x+1)}$

Given the fraction $\frac{3x-1}{(x-1)(x+1)}$ Can we decompose into partial fractions whose sum equals this fraction?

$$\frac{3x-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}$$

$$3x-1 = A(x+1)\ B(x-1)$$

$x = -1 \Rightarrow -4 = 0 - 2B$

$\boxed{B = 2}$

$x = 1 \Rightarrow 2 = 2A$

$A = 1$

Method 2

$3x-1 = A(x+1) + B(x-1)$

$3x-1 = Ax + A + Bx - B$

$3x-1 = x(A+B) + A - B$

$3 = A + B$

$A - B = -1 \quad +$

$2A = 2$

$\boxed{A = 1}$

$\boxed{B = 2}$

$* \int \frac{3x-1}{x^2-1} \, dx = \int \frac{3x-1}{(x-1)(x+1)} \, dx$

$= \int \frac{1}{x-1} + \frac{2}{x+1} \, dx$

$= \ln|x-1| + 2\ln|x+1| + C$

$$\int \frac{dx}{x^3 - x} = \int \frac{dx}{x(x-1)(x+1)}$$

$$\frac{1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$1 = A(x-1)(x+1) + B(x)(x+1) + C(x)(x-1)$$

$x = 0 \Rightarrow 1 = -A$

$\boxed{A = -1}$

$x = 1 \Rightarrow 1 = 0 + 2B + 0$

$\boxed{B = \frac{1}{2}}$

$x = -1 \Rightarrow 1 = 0 + 0 + 2C$

$\boxed{C = \frac{1}{2}}$

$$= \int \frac{-1}{x} + \frac{\frac{1}{2}}{x-1} + \frac{\frac{1}{2}}{x+1}\, dx$$

$$= -\ln|x| + \frac{1}{2}\ln|x-1| + \frac{1}{2}\ln|x+1| + C$$

$$* \int \frac{dx}{(x-1)(x^2-1)}$$

$$\frac{1}{(x-1)(x-1)(x+1)} = \frac{1}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$$

$$\frac{1}{(x-1)^2(x+1)} = \frac{A(x-1)(x+1) + B(x+1) + C(x-1)^2}{(x-1)^2(x+1)}$$

$$1 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$$x=1 \Rightarrow 1 = 0 + 2B + 0$$

$$\boxed{B = \frac{1}{2}}$$

$$x=-1 \Rightarrow 1 = 0 + 0 + 4C$$

$$\boxed{C = \frac{1}{4}}$$

$$x=0 \Rightarrow 1 = -A + B + C \Rightarrow 1 = -A + \frac{1}{2} + \frac{1}{4}$$

$$A = \frac{3}{4} - \frac{4}{4}$$

$$\boxed{A = \frac{-1}{4}}$$

$$= \int \frac{-\frac{1}{4}}{(x-1)} + \frac{\frac{1}{2}}{(x-1)^2} + \frac{\frac{1}{4}}{x+1}$$

$$= \frac{-1}{4}\ln(x-1) + \frac{1}{2}\left(\frac{-1}{x-1}\right) + \frac{1}{4}\ln|x+1| + C$$

Ex Decompose into partial fractions

$$\frac{x+1}{(x-1)^3(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{x+1}$$

ونكمل

$$\frac{1}{(x-1)^n(x+1)} = \frac{A_1}{(x-1)} + \frac{A_2}{(x-1)^2} + \dots + \frac{A_n}{(x-1)^n} + \frac{C}{x+1}$$

* $\int \frac{f(x)}{g(x)}\, dx$ → both $f(x)$ and $g(x)$ are poly. and the degree of $f(x) < g(x)$

such integral can be solved by partial fraction as follows

(1) we factor out $g(x)$ into lines factors and ... factor

(2) we put together simeler factor

(3) we decompose the fraction $\frac{f(x)}{g(x)}$ as partial fraction

Ex $\frac{f(x)}{(ax+b)^n g(x)} = \frac{A_1}{(ax+b)} + \frac{A_2}{(ax+b)^2} + \cdots + \frac{A_n}{(ax+b)^n} + \frac{B}{g(x)}$

Ex $\frac{f(x)}{(ax^2+bx+c)^n g(x)} = \frac{A_1 x + B_1}{(ax^2+bx+c)} + \frac{A_2 x + B_2}{(ax^2+bx+c)^2} + \cdots + \frac{A_n x + B_n}{(ax^2+bx+c)^n} + \frac{C}{g(x)}$

!! لا يتحلل

* Decompose The following fractions as partial fractions.

(1) $\dfrac{x+5}{(x-1)(x^2-1)} = \dfrac{x+5}{(x-1)(x-1)(x+1)} = \dfrac{x+5}{(x-1)^2(x+1)} = \dfrac{A}{(x-1)} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x+1)}$

(2) $\dfrac{x^2+10}{x^3+x} = \dfrac{x^2+10}{x(x^2+1)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$

(3) $\dfrac{x^3+1}{x(x-5)^2(x^2+x+1)^2} = \dfrac{A}{x} + \dfrac{B}{(x-5)} + \dfrac{C}{(x-5)^2} + \dfrac{Dx+E}{(x^2+x+1)} + \dfrac{Fx+G}{(x^2+x+1)^2}$

\# $\displaystyle\int \frac{dx}{x(x^2+x+1)}$

$\dfrac{1}{x(x^2+x+1)} = \dfrac{A}{x} + \dfrac{Bx+C}{(x^2+x+1)}$

$1 = Ax^2 + Ax + A + Bx^2 + Cx$

$x = 0 \rightarrow \boxed{A = 1}$

$B = -1$

$C = -1$

$\displaystyle\int \frac{1}{x} + \frac{-x-1}{x^2+x+1}\, dx$

$\displaystyle = \ln|x| - \int \frac{x+1}{x^2+x+1}\, dx$

$\displaystyle = \ln|x| - \frac{1}{2}\int \frac{2x+1+1}{x^2+x+1}\, dx$

$\displaystyle = \ln|x| - \frac{1}{2}\int \frac{2x+1}{x^2+x+1}\, dx + \int \frac{1\,dx}{x^2+x+1}$

$\displaystyle = \ln|x| - \frac{1}{2}\ln|x^2+x+1| + \int \frac{1}{(x+\frac{1}{2})^2 + \frac{3}{4}}\, dx$

$\displaystyle = \ln|x| - \frac{1}{2}\ln|x^2+x+1| - \frac{1}{2}\left(\frac{2}{\sqrt{3}}\right)\tan^{-1}(x) + \frac{x+}{\frac{\sqrt{3}}{2}}$

$\int \frac{x+1}{x^4+x^2}\,dx$

$$\frac{x+1}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$$

$$x+1 = A(x)(x^2+1) + B(x^2+1) + (Cx+D)(x^2)$$

for $x=0 \rightarrow \boxed{B=1}$

$$x+1 = x^3[A+C] + x^2[B+D] + xA + B$$

$\boxed{A=1}$

$B+D=0$

$\boxed{D=-1}$

$A+C=0$

$\boxed{C=-1}$

$$= \int \frac{1}{x} + \frac{1}{x^2} - \frac{x-1}{x^2+1}\,dx$$

$$= \ln|x| + \frac{-1}{x} - \frac{1}{2}\int \frac{2x+1+1}{x^2+1}\,dx$$

$$= \ln|x| - \frac{1}{x} - \frac{1}{2}\int \frac{2x}{x^2+1} - \int \frac{2}{x^2+1}\,dx$$

$$= \ln|x| - \frac{1}{x} - \frac{1}{2}\ln|x^2+1| + 2\tan^{-1}x + C$$

* $\int \frac{(\sin x + 3) \cos x \, dx}{\sin^3 x + 3\sin^2 x + 3\sin x + 1}$

$= \int \frac{y+3}{y^3 + 3y^2 + 3y + 1} dy$

$y = \sin x$

$dy = \cos x \, dx$

$\int \frac{y+3}{(y+1)^3} dy$

$y + 1 = u$

$du = dy$

$= \int \frac{u+2}{u^3} du$

$= \int \frac{1}{u^2} du + 2\int \frac{du}{u^3}$ ⟿ partial fraction.

$= -\frac{1}{u} +$

* Consider

$$* \int \frac{dx}{x+x^{100}} = \int \frac{dx}{x^{100}(x^{-99}+1)}$$

$y = x^{-99}+1$

$dy = -99x^{-100}\,dx$

$$= \int \frac{dy}{-99y} = \frac{-1}{99}\ln|y| + C$$

$$= \frac{-1}{99}\ln|x^{-99}+1| + C$$

Rationalization ‼ ◡

$$* \int \frac{dx}{1+\sqrt[3]{x}}$$

$x = y^3$

$dx = 3y^2\,dy$

$y = \sqrt[3]{x}$

$$= \int \frac{3y^2}{1+y}\,dy$$

$$\begin{array}{r} 3y-3 \\ y+1 \enclose{longdiv}{3y^2} \\ -3y^2+3y \\ \hline -3y \\ +3y+3 \\ \hline 3 \end{array}$$

$$= \int 3y-3+\frac{3}{1+y}\,dy$$

$$= \frac{3}{2}y^2-3y+3\ln|1+y|+C$$

$$= \frac{3}{2}\sqrt[3]{x^2}-3\sqrt[3]{x}+3\ln|1+\sqrt[3]{x}|+C$$

$$*\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$$

$y = \sqrt[6]{x}$

$x = y^6$

$$\int \frac{6y^5}{y^3+y^2}\, dy$$

$dx = 6y^5\, dy$

$$\int 6y^2 - 6y + 6 - \frac{6y^2}{y^2(y+1)}\, dy$$

$$\begin{array}{r} 6y^2 - y + 1 \\ y^3+y^2 \overline{\big) 6y^5} \\ \underline{-y^5 \mp y^4} \\ -y^4 \\ \underline{\pm y^4 + y^3} \\ y^3 \\ \underline{-y^3 \mp y^2} \\ -y^2 \end{array}$$

$$= \frac{6y^3}{3} - \frac{6y^2}{2} + 6y - 6\ln|y+1| + C$$

$$= 2y^3 - 3y^2 + 6y - 6\ln|y+1| + C$$

$$= 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\ln|\sqrt[6]{x}+1| + C$$

$$*\int \sqrt{4-\frac{1}{x}}$$

$$4-\frac{1}{x}=y^2$$

$$x=\frac{1}{4-y^2}$$

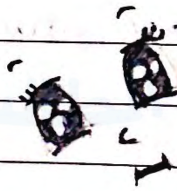
$$dx=\frac{-2y}{(4-y^2)^2}\,dy$$

$$=\int \frac{y\cdot 2y}{(4-y^2)^2}\,dy \qquad =\int \frac{2y^2}{(4-y^2)^2}$$

$$\frac{2y^2}{(4-y^2)^2}=\frac{2y^2}{(2-y)^2(2+y)^2}=\frac{A}{2-y}+\frac{B}{(2-y)^2}+\frac{C}{(2+y)}+\frac{D}{(2+y)^2}$$

$$2y^2=A(2-y)(2+y)^2+B(2+y)^2+C(2-y)^2(2+y)+D(2-y)^2$$

Continue



* $\int \sqrt{4-\frac{1}{x}}\ dx$

$\int \frac{\sqrt{4x-1}}{\sqrt{x}}$

$4x-1=y^2$

$4x=y^2+1$

$x=\frac{1}{4}y^2+\frac{1}{4}$

$dx=\frac{1}{2}y\,dy$

$\int \frac{y \cdot \frac{1}{2}y}{\frac{1}{2}\sqrt{y^2+1}}\ dy$

$= \int \frac{y^2}{\sqrt{y^2+1}}\ dy$

$y=\tan\theta$

$dy=\sec^2\theta\ d\theta$

$= \int \frac{\tan^2\theta}{\sec\theta}\ \sec^2\theta\ d\theta$

$= \int (\sec^2\theta-1)\sec\theta\ d\theta$

$\int \sec^3\theta - \sec\theta\ d\theta$

$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta| - \ln|\sec\theta+\tan\theta| + C$

Half Angle Subsitution.

Consider

$$\int \frac{dx}{1+\cos x} = \int \frac{1-\cos x}{(1+\cos x)(1-\cos x)}\,dx = \int \frac{1-\cos x}{1-\cos^2 x}\,dx$$

$$= \int \frac{1-\cos x}{\sin^2 x}\,dx = \int \frac{1}{\sin^2 x}\,dx - \int \frac{\cos x}{\sin^2 x}\,dx$$

$$= -\cot x + \frac{1}{\sin x} + C$$

$$* \int \frac{dx}{2+\cos x}$$

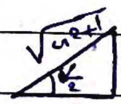
$u = \tan\left(\frac{x}{2}\right)$ تعويض بنصف الزاوية

$\frac{x}{2} = \tan^{-1} u$

$\frac{dx}{2} = \frac{1}{1+u^2}\,du$

$dx = \frac{2}{1+u^2}\,du$

$$\int \frac{1}{2+\frac{1-u^2}{1+u^2}} * \frac{2}{1+u^2}\,du$$



$$= \int \frac{\cancel{1+u^2}}{2+2u^2+1-u^2} * \frac{2}{\cancel{1+u^2}}\,du$$

$\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1+u^2}}$

$$\int \frac{2}{u^2+3}\,du = 2\int \frac{du}{u^2+3}$$

$\sin\left(\frac{x}{2}\right) = \frac{u}{\sqrt{1+u^2}}$

$$= \frac{2\tan^{-1}\left(\frac{u}{\sqrt{3}}\right)}{\sqrt{3}} + C$$

$\sin x = \frac{2u}{1+u^2}$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) + C$$

$\cos x = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$

$= \frac{1}{1+u^2} - \frac{u^2}{1+u^2}$

$\cos x = \frac{1-u^2}{1+u^2}$

$$\star \int \frac{dx}{1+\sin x+\cos x}$$ ← مثل السؤال السابق

$u = \tan\left(\frac{x}{2}\right)$

$\frac{x}{2} = \tan^{-1} u$

$\frac{dx}{2} = \frac{1}{1+u^2}\, du$

$$\int \frac{\frac{2du}{1+u^2}}{1+\frac{2u}{1+u^2}+\frac{1-u^2}{1+u^2}}$$

$$\int \frac{\frac{2du}{\cancel{1+u^2}}}{\frac{1+u^2+2u+1-u^2}{\cancel{1+u^2}}} = \int \frac{2du}{2(u-1)} = \ln|u-1| + C$$

$$= \ln\left|\tan\left(\frac{x}{2}\right)-1\right| + C$$

$$\star \int \sqrt{\frac{1+x}{1-x}}$$

$\frac{1+x}{1-x} = y^2$

$1+x = y^2 - y^2 x$

$y^2 - 1 = x + y^2 x$

$x(1+y^2) = y^2 - 1$

$x = \frac{y^2-1}{1+y^2}$

$dx = \frac{(1+y^2)2y-(y^2-1)2y}{(1+y^2)^2}$

$dx = \frac{4y}{(1+y^2)^2}\, dy$

$$= \int \frac{y \cdot 4y}{(1+y^2)^2}\, dy$$

$y = \tan\theta$

$dy = \sec^2\theta\, d\theta$

$$= \int \frac{4\tan^2\theta}{\sec^{\cancel{4}\,2}\theta}\, \cancel{d\theta}\, \cancel{\sec^2\theta}\, d\theta$$

$$\int \frac{4\tan^2\theta}{\sec^2\theta}\, d\theta$$

$$= \int \frac{4\sin^2\theta}{\cos^2\theta} * \cancel{\cos^2\theta}\, d\theta$$

Continue

Improper Integral !!

Consider the integral

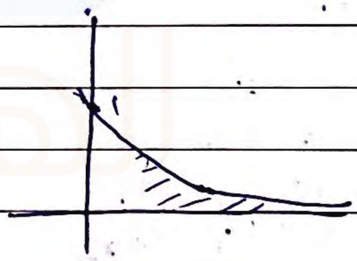
$$\int_a^b f(x)\,dx$$

* if $\underline{a}$ or $\underline{\underline{b}}$ is $\mp\infty$ then the integral is called an improper integral of the first kind

* if $f(x)$ is undefined on $[a,b]$, then the integral is called an improper integral of the second kind.

* $$\int_0^\infty \frac{dx}{1+x^2} \qquad f(x) = \frac{1}{1+x^2}$$

$$\lim_{b \to \infty} \int_0^b \frac{1}{1+x^2}\,dx$$



$$= \lim_{b \to \infty} \tan^{-1} x \Big]_0^b$$

* لو طلع عدد ← المساحة محدودة ولها حد هذا العدد
* لو ما طلع عدد ← غير محدودة

$$= \lim_{b \to \infty} \tan^{-1} b - \tan^{-1}(0)$$

$$= \frac{\pi}{2} - 0 = \frac{\pi}{2} \qquad \text{Convergent}$$

$$* \int_{-\infty}^{1} e^{-x}\, dx$$

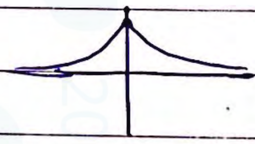
$$\lim_{a \to -\infty} \int_{a}^{1} e^{-x}\, dx$$

$$\lim_{a \to -\infty} -e^{-x} \Big]_{a}^{1}$$

$$\lim_{a \to -\infty} -e^{-1} + e^{-a}$$

$$= -\frac{1}{e} - \infty = \infty$$

$$* \int_{-\infty}^{\infty} \frac{dx}{1+x^2}$$



$$= \int_{-\infty}^{0} \frac{dx}{1+x^2} + \int_{0}^{\infty} \frac{dx}{1+x^2}$$

حسبناه في السؤال الأول $= \frac{\pi}{2}$ Convergent

$$= \lim_{a \to -\infty} \int_{a}^{0} \frac{dx}{1+x^2}$$

$$\lim_{a \to -\infty} \tan^{-1} x \Big]_{a}^{0}$$

$$\tan^{-1}(0) - \tan^{-1}(-\infty)$$

$$= 0 - \left(-\frac{\pi}{2}\right) = \frac{\pi}{2}$$

$$\frac{\pi}{2} + \frac{\pi}{2} = \pi = \int_{-\infty}^{\infty} \frac{dx}{1+x^2}$$ Convergent

$$\ast \quad \int_0^1 \frac{dx}{x^{\frac{1}{2}}}$$

$f(x) = \frac{1}{\sqrt{x}}$

$0 \quad + + + \quad 1$

$$\lim_{a \to 0^+} \int_a^1 \frac{dx}{\sqrt{x}}$$

غير معرف عند الصفر

$$\lim_{a \to 0^+} 2\sqrt{x}\,\Big]_a^1$$

$$\lim_{a \to 0^+} 2\sqrt{1} - 2\sqrt{a}$$

$$2 - 0 = 2$$ Convergent

$$\ast \quad \int_0^2 \frac{dx}{x^2 - 4}$$

$\frac{1}{x^2 - 4}$

$-2 \quad 0 \quad + + + \quad 2$

$$\lim_{b \to 2^-} \int_0^b \frac{dx}{x^2 - 4} \rightsquigarrow$$ by partial fraction

$$\lim_{b \to 2^-} \int_0^b \frac{1}{4}\left(\frac{1}{x-2} + \frac{1}{x+2}\right)$$

$$\lim_{b \to 2^-} \frac{1}{4} \ln|x-2| - \frac{1}{4} \ln|x+2| \Big]_0^b$$

$$\lim_{b \to 2^-} \frac{1}{4} \left[\ln\left|\frac{x-2}{x+2}\right|\right]_0^b$$

$$= \lim_{b \to 2^-} \frac{1}{4} \left[\ln\left|\frac{b-2}{b+2}\right| - \frac{1}{4} \ln|0|\right.$$

$$= -\infty - 0 = -\infty$$ divergent

$$\int_0^2 \frac{dx}{x-1} = \underbrace{\int_0^1 \frac{dx}{x-1}}_{I} + \underbrace{\int_1^2 \frac{dx}{x-1}}_{II}$$

(I)

$$\int_0^1 \frac{dx}{x-1} = \lim_{b \to 1^-} \int_0^b \frac{dx}{x-1} = \lim_{b \to 1^-} \ln|x-1| \Big]_0^b$$

$$= \lim_{b \to 1^-} \ln|b-1| - \ln|1|$$

$$= -\infty - 0 \quad \text{divergent}$$

so

$$\int_1^2 \frac{dx}{x-1} \quad \text{divergent}$$

$$* \int_0^\infty \frac{dx}{\sqrt{x}(1+x)}$$

$$= \int_0^1 \frac{dx}{\sqrt{x}(1+x)} + \int_1^\infty \frac{dx}{\sqrt{x}(1+x)}$$

I II

I $\int_0^1 \frac{dx}{\sqrt{x}(1+x)} = \lim_{a \to 0^+} \int_a^1 \frac{dx}{\sqrt{x}(x+1)}$

$x = y^2$

$dx = 2y\,dy$

$x = a \to y = \sqrt{a}$

$x = 1 \to y = 1$

$$= \lim_{a \to 0^+} \int_{\sqrt{a}}^1 \frac{2y\,dy}{y(y^2+1)}$$

$$= \lim_{a \to 0^+} 2\tan^{-1} y \Big]_{\sqrt{a}}^1$$

$$= \lim_{a \to 0^+} 2\tan^{-1}(1) - 2\tan^{-1}(\sqrt{a})$$

$$= 2\frac{\pi}{4} - 2\tan^{-1}(0)$$

$$= \frac{\pi}{2} - 2(0) = \frac{\pi}{2}$$ Convergent

II $\int_1^\infty \frac{dx}{\sqrt{x}(1+x)} = \lim_{b \to \infty} \int_1^b \frac{dx}{\sqrt{x}(1+x)}$

$x = y^2$

$dx = 2y\,dy$

$x = 1 \to y = 1$

$x = b \to y = \sqrt{b}$

$$= \lim_{b \to \infty} \int_1^{\sqrt{b}} \frac{2dy}{y^2+1}$$

$$= \lim_{b \to \infty} 2\tan^{-1}(\sqrt{b}) - 2\tan^{-1}(1)$$

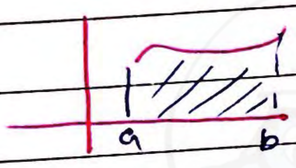
$$= \frac{2\pi}{2} - \frac{2\pi}{4} = \frac{\pi}{2}$$ Convergent

$$\int_0^\infty \frac{dx}{\sqrt{x}(x+1)} = \frac{\pi}{2} + \frac{\pi}{2} = \pi \ \#$$

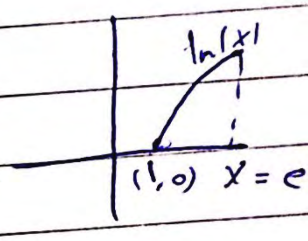
CH 6 : Area between 2 curves ...

* Area and integration

1. $f(x) \geq 0$ on $[a,b]$, then the Area $= \int_a^b f(x)\,dx$.



Ex Find the area of the shaded region

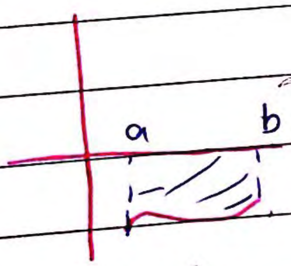


$$A = \int_1^e \underbrace{\ln|x|}_{u}\,\underbrace{dx}_{dv}$$

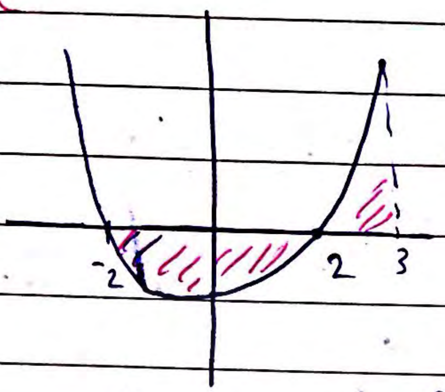
$$= x\ln|x| - x\Big]_1^e = -\ln(1) - 1 + \ln(e) - e$$

$$= 1$$

2. If $f(x) \leq 0$ on $[a,b]$, then the Area $= -\int_a^b f(x)\,dx$



Ex



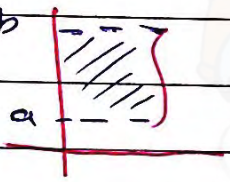
$f(x) = x^2 - 4$

$\to x^2 - 4 = 0$

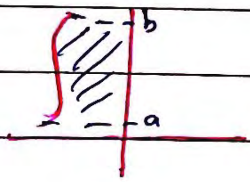
$x = \mp 2$

$$A = -\int_{-2}^{2} x^2 - 4\,dx + \int_{2}^{3} x^2 - 4\,dx$$

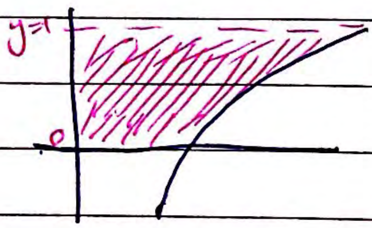
3. If $x = f(y) \geq 0$ on $[a,b]$, then Area $= \int_a^b f(y)\,dy$ بالنسبة y



4. If $x = f(y) \leq 0$ on $[a,b]$ then Area $= -\int_a^b f(y)\,dy$



Ex find the area of the shaded region:



$x = e^y$

$$\to A = \int_0^1 e^y\,dy$$

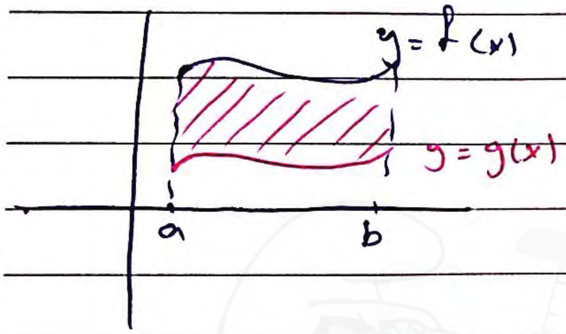
$$= e^y \Big]_0^1 = e - 1$$

* Area between Curves:-

→ Given, $y = f(x)$ and $y = g(x)$,, on $[a, b]$, then

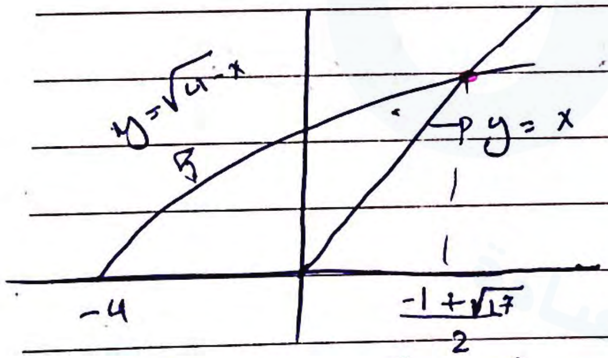
the area between the 2 Curves $= \int_a^b f(x) \, g(x) \, dx$

العلوي - السفلي



→ Area $= \int_a^b f(x) - g(x) \, dx$

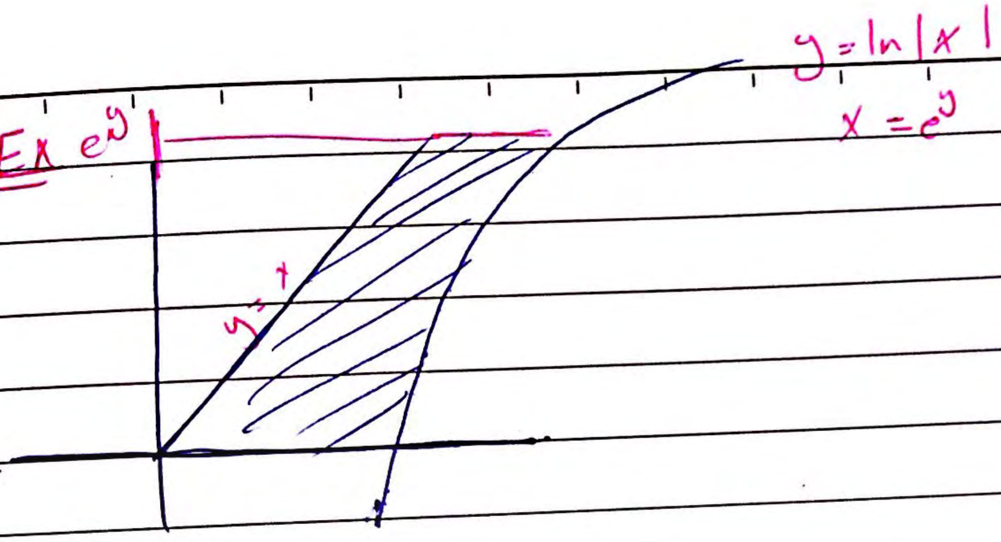
Ex Find the area of the shaded region



* $\sqrt{4-x} = x \longrightarrow x^2 + x - 4 \longrightarrow \frac{-1 \mp \sqrt{17}}{2}$

$$A = \int_{-4}^{0} \sqrt{4-x} + \int_{0}^{\frac{-1+\sqrt{17}}{2}} \sqrt{4-x} - x \, dx$$

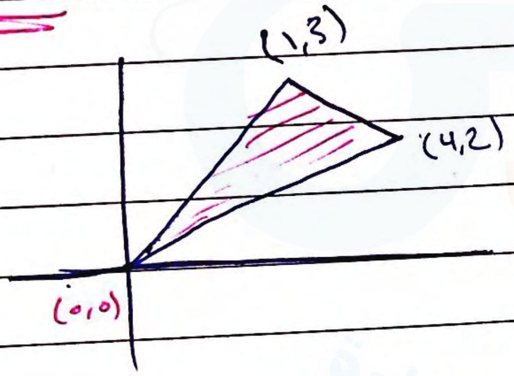
Ex e^y



$$A = \int_0^e e^y - y \, dy$$

$$= e^y - \frac{y^2}{2} \Big]_0^e$$

H.w



نخرج معادلة الخط المستقيم لكل خط منهم
ونجد المساحة

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